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Axiomatic Theories of Truth, Bounded Induction and Reflection Principles

Abstract

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The main objective of our thesis is to study properties of the notion of truth. Staying within the tradition initiated by Tarski, we take truth to be a property of sentences and apply formal tools in order to clarify and answer interesting us questions. To be more precise, our study consists of verifying how various properties of truth relate to each other. We may think of a property of a notion P as of a linguistic law governing its use. Stating this a little bit more formally, they may be considered meaning postulates or *axioms* of our language that regulate the assertability conditions for sentences in which P occurs. In the dissertation we are interested in the case when P is the notion of truth and we investigate various *axiomatic theories of truth* which we take to model possible meaning postulates for this notion.

In our thesis we focus primarily on compositional, stratified theories of truth. Such theories admit axioms saying how the truth of a sentence depends on the truth of its immediate subformulae (compositionality) but the truth is defined only for sentences that do not themselves contain the truth predicate (stratification). We concentrate on the case when the object language is the language of Peano Arithmetic and treat the latter as the object theory.

The properties of the notion of truth we study are listed in the title of our dissertation. The first one is weak inductiveness: we investigate which properties of truth follow when we assume that every bounded formula with the predicate T satisfies induction. The question is intriguing for at least two reasons. Firstly, with restricted possibilities of reasoning by induction, it is highly non-obvious whether one can invoke standard proof techniques (such as induction on the build-up of formulae or induction on the length of proofs), which are normally used in order to demonstrate that the notion of truth has certain properties. Secondly, it is also highly unobvious what sort of sentences from the object language will be provable in such a restricted setting, with only a weakly inductive notion of truth at our disposal.

The thesis contains original results on weakly inductive axiomatic truth theories, as described in the last paragraph. We prove that such basic axioms suffice to guarantee that the notion of truth is very well-behaved, meaning that it enjoys many further natural properties (such as closure under reasoning in First-Order Logic, in the sense of the object theory). Moreover, we show that this is the case for theories of both classically and non-classically compositional truth.

The last place on the list from the title of our dissertation is occupied by *reflection principles*. Intuitively, a reflection principle for a set of sentences X expresses the soundness of X with respect to some logic \mathcal{L} , from the perspective of the theory for which this principle is stated. Having the truth predicate at our disposal, we may express directly the above intuition in a single sentence, for example,

For every sentence ϕ of \mathcal{L} , if ϕ is provable from X in logic \mathcal{L} , then ϕ is true.

We distinguish *completeness* and *closure* reflection principles, depending on whether X contains the set of all true sentences (the latter type) or not (the former one). Let us note that a *closure* reflection principle for a logic \mathcal{L} implies that the property of being true is preserved in reasoning in \mathcal{L} ; or, to put it differently, that the set of true sentences is *closed* under reasoning in \mathcal{L} . One of the main objectives of our thesis is to determine how such principles relate to the bounded induction for the truth predicate, if the latter is compositional. In particular, we ask whether bounded induction permits us to prove reflection principles, and if so, which forms of reflection become provable as soon as the bounded induction is added to our truth theory. Moreover, we investigate whether this relation depends on how (classically or non-classically) compositional the truth predicate is. The thesis contains new results clarifying these dependencies.

Generally speaking, we are interested in the *strength* of the truth principles. We introduce three different formal explications of this notion, but are focused mainly on the *proof-theoretical one*: our aim is to characterize the sets of sentences of the object language \mathcal{L} , which can be deduced from various combinations of the truth properties we consider.